Bob's TechTalk #28
by Bob Eckweiler, AF6C

Capacitors - Part III of IV

Phase Shift and Power

Last month I said we were going to explore phase shift and power factor this month, Before we do that, perhaps gaining an understanding of how power, voltage and current are measured in an AC circuit is in order. So this month we'll look first at how AC is measured. Sine wave AC voltages and currents can be measured as either peak or peak-to-peak values. See figure 1. A peak measurement is made from the zero line to one of the peak points on the sine wave. A peak-to-peak measurement is made between the negative peak and the positive peak of the sine wave. It should be obvious from figure 1 that the peak-to-peak measurement is twice the peak measurement. But what's the average voltage or current? Since the voltage or current is symmetrical around the zero-line, the average is zero. That's pretty useless! Try telling someone who's been shocked by high voltage AC and lived, that the average voltage was zero and you don't see the 'big deal'!

Since the average voltage and current of a sine wave is zero, engineers - the clever people that they are - came up with a way to measure "effective" voltage instead of average voltage. The effective voltage and current is the voltage and current values that, if used in a DC circuit, would produce the same average power. This effective voltage and current is called the RMS voltage and current. RMS stands for *root mean square* and is one of those terms that help engineers command high salaries. The RMS voltage and RMS current of a sine wave are just the peak value divided by the square root of two or 0.707 times the peak.

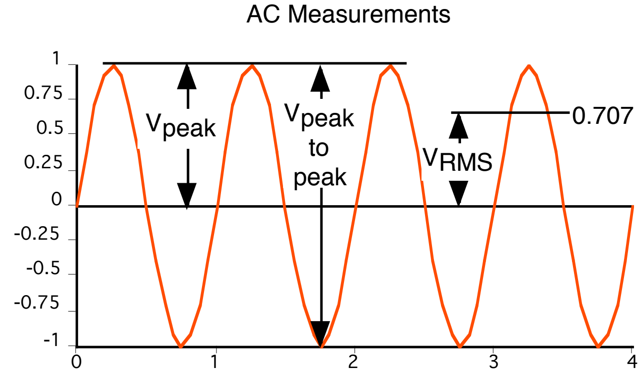


Figure 1 – How peak and peak-to-peak voltages are measured on an AC sine wave. Current measurements are similar.

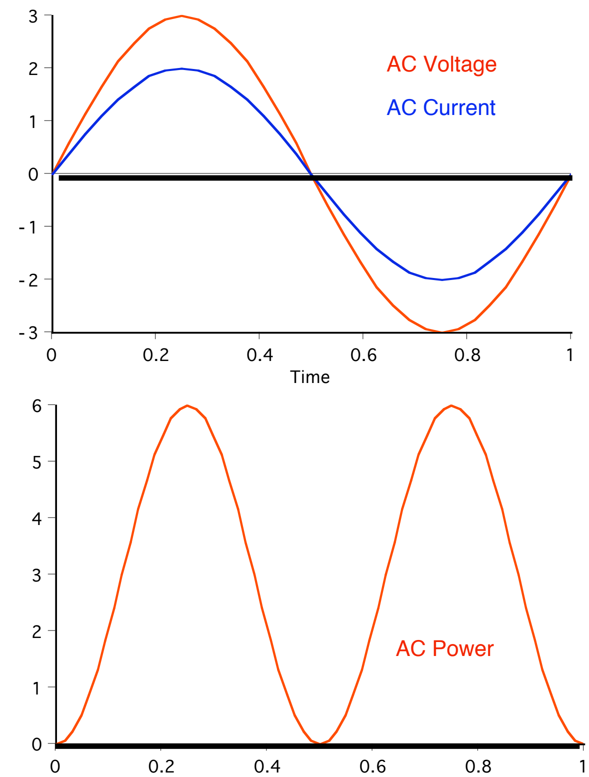


Figure 2 – AC Voltage and Current in a Resistive circuit and the resulting AC power

If you venture to look at a little math and Ohm's law, here's how that is derived. First, let's look at the average power of a sine wave. The top part of figure 2 shows a 3 volt peak sine wave and 2 ampere peak sine wave (as one might see if that voltage was fed across a 1.5Ω resistor.) The power at any given time is the voltage multiplied by the current at that time. This is right from Ohm's law. The bottom part of figure 2 shows the resulting power. You can check a few points yourself (Remember that multiplying two negative numbers results in a positive number.) Note that the power is a sine wave also, but with half the wavelength (twice the frequency) and it is always on the positive side of zero. From the figure you can also surmise that the average power in an AC circuit is half of the peak power, or 3 watts.

$$P_{AVG} = \frac{P_{PEAK}}{2}$$

Now let's define an *effective voltage* and *effective current* such that they will meet Ohm's law in the following three forms:

$$V_{EFF} \cdot I_{EFF} = P_{AVG}$$

$$I_{EFF}^2 \cdot \mathrm{R} = P_{AVG}$$

$$\frac{V_{EFF}{}^2}{\mathrm{R}} = P_{AVG}$$

Next, we have to find how the effective voltage and current relate to the peak voltage and current.

$$P_{PEAK} = \frac{V_{PEAK}{}^2}{R} = 2 \cdot P_{AVG} = \frac{2 \cdot V_{EFF}{}^2}{R}$$

If you look at the two terms with R in them you'll see that:

$$\frac{2 \cdot V_{EFF}{}^2}{R} = \frac{V_{PEAK}{}^2}{R}$$

This solves to:

$$V_{EFF} = \frac{1}{\sqrt{2}} \cdot V_{PEAK}$$

or

$$V_{EFF} = 0.707 V_{PEAK} = V_{RMS}$$

The effective voltage is also called the RMS (root mean square) voltage; it's the value that is commonly used for measuring AC voltage. In the US your wall socket AC voltage is typically 117V rms. This relates to 166 Vpeak or 332 Vpeak-to-peak). As mentioned earlier: *the rms. value of the voltage and current are just the equivalent value that would produce the same average power in a DC circuit as the peak values would in an AC circuit*. From now on, when we talk of a voltage or current in an AC circuit we'll mean the RMS (effective) value unless stated otherwise.

Power in a Capacitor:

Last month we saw that when a sine wave voltage is placed across a capacitor the current is also a sine wave, but is shifted by 90°. Figure 3 is identical to figure two except that the current has been shifted to lead the voltage by 90° as was shown last month in figure 4. Note that the resulting power is similar to figure 2 except that the sine wave now lies centered on the zero axis. The capacitor is taking power from the circuit for half of the time and returning it to the circuit during the other half. This means that the average power in a capacitance is zero and the (ideal) capacitor dissipates no power from the circuit.

If a resistance is placed in series with a capacitor and a sine wave voltage is applied, what happens to the current then? Looking at figures 2 and 3 you can surmise that the current will also have a positive phase shift (lead the voltage) somewhere between the 0° shift of figure 2 and the 90° shift of figure 3; and that the amount of this shift depends upon the resistance and the capacitor's reactance. You'd be correct! The phase shift angle's tangent is just the reactance over the resistance. Or:

$$\tan\theta = \frac{X_C}{R}$$

If you're familiar with trigonometry, you'll immediately see that when Xc is zero, the phase angle zero and when the resistance is zero the phase angle is 90°, just as the graphs depict.

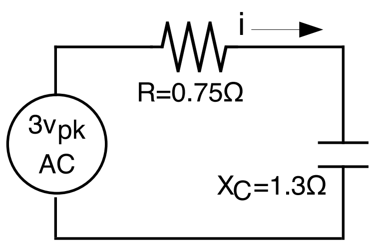


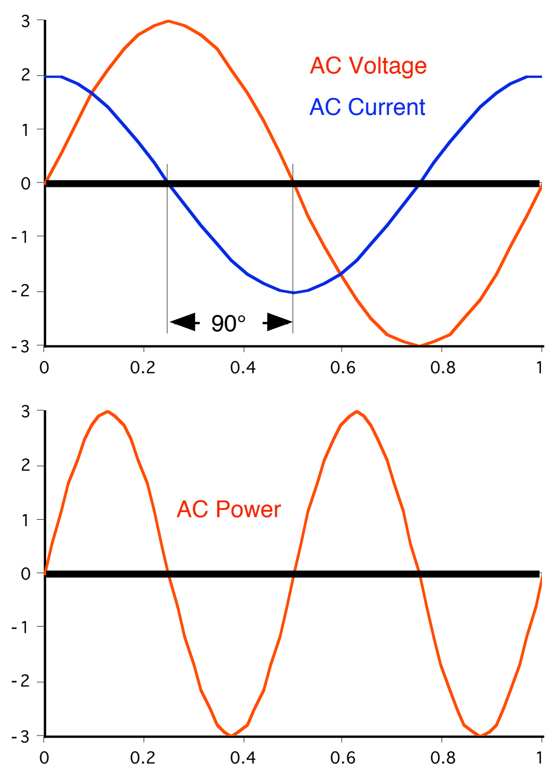
Figure 5 – Circuit for Figure 4 graph

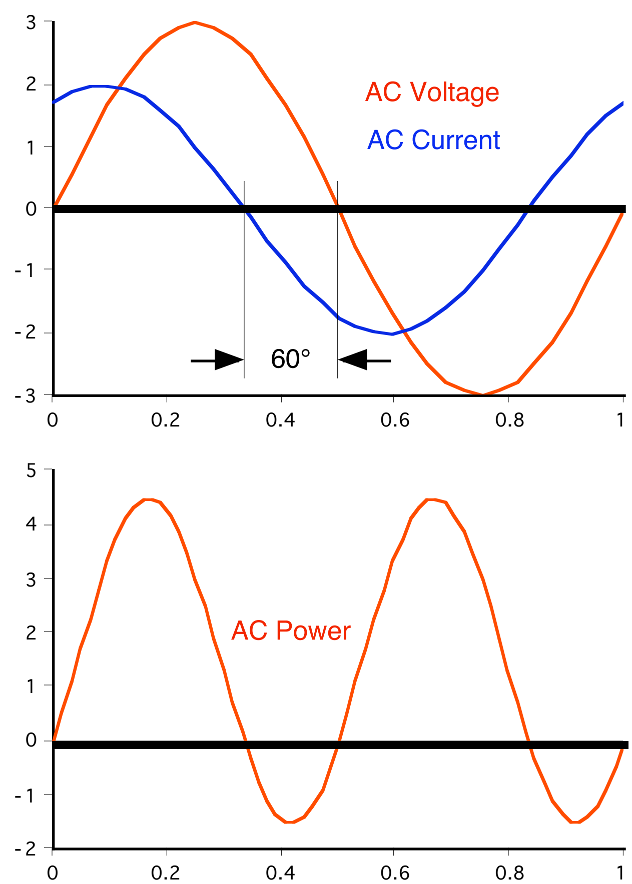
Let's look at figure 4. It's similar to figures 2 and 3, and shows the relationship of the voltage and current of the circuit shown in figure 5. The current is leading the voltage by a phase shift of 60°. Note that the power graph is shifted part-way between where it is in figures 2 and 3. Some of the power is being dissipated in the resistor and some is being stored and returned by the capacitor. What percent of the power is being dissipated by the resistor? The formula is:

$$P_R = V \cdot I \cdot \cos\theta$$

Theta (the oh with the line through it) is the symbol for the phase angle. Don't fret if your trigonometry is rusty; if theta is 60°, the cosine of 60° is 0.5 and the power in the resistor is:

$$P_R = (0.707 \cdot 3) \cdot (0.707 \cdot 2) \cdot 0.5$$
$$= 1.5 \text{ watts}$$





Remember that the voltage and current are peak values and each must be multiplied by 0.707 to get their effective or RMS value.

We've covered a lot; and used more math than I'd have liked. If the math confuses you, try to read the text and examine the graphs to get the concept. Next month we'll finally get to power factor in a capacitor and also power factor in your house electrical system. They're two different "enchiladas", but both with similar theory.

73, from AF6C

This article is based on the TechTalk article that originally appeared in the April 2004 issue of RF, the newsletter of the Orange County Amateur Radio Club - W6ZE.