

Bob's TechTalk #12
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Ohm's Law (Part II of IV):

Thévenin's Theorem:

Last month the three forms of Ohm's Law were introduced. For simple circuits the law is easy to apply, as we saw in the examples and problems. However, as the last problem emphasized, sometimes a circuit can present a problem that is less intuitive to solve. Two theorems may be used to simplify more complex circuits; this month we're going to look at Thévenin's Theorem. The theorem basically states that: *Any two terminal linear circuit composed of resistors and voltage sources may be replaced by one power source in series with a fixed resistor.* Whoa! What does that really mean?

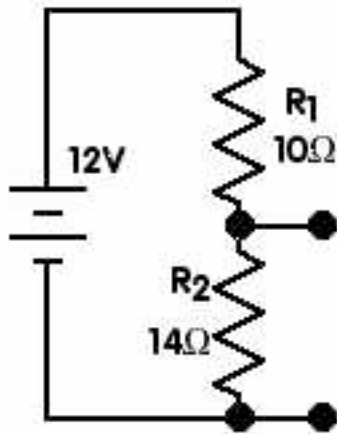
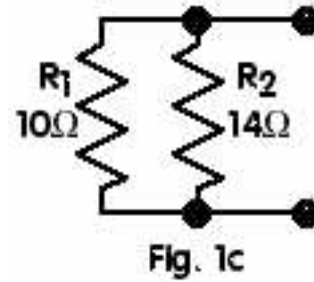


Fig. 1a

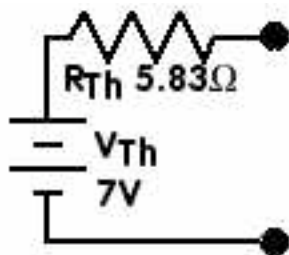


Fig. 1b

Let's look at the voltage divider circuit from last month (see Figure 1a). Two output terminals have been added and the meters removed; otherwise it is the same circuit as figure 2 from last month. According to Thévenin's Theorem this circuit may be replaced with the circuit in figure 1b. The Thévenin's equivalent voltage source is the calculated open circuit voltage across the two terminals. The Thévenin's equivalent resistance may be calculated by assuming all the voltage sources are replaced by short circuits and then calculate the resistance between the two terminals.

First let's calculate the Thévenin's equivalent voltage source. Last month we used Ohm's Law to find the current flowing through the resistors and used it again to find the voltage across R₂ to solve the voltage divider. You can also use the Voltage Divider equation (6) to find the Thévenin's equivalent voltage:

$$V_{Th} = \left(\frac{R_2}{R_1 + R_2} \right) \times V \quad (6)$$

The numerator in this equation is always the (series) value of the resistor(s) across the output terminals (in this case R₂) and the denominator is the total series resistance of the circuit (R₁ + R₂ in this case). Solving figure 1a we get:

$$V_{Th} = \left(\frac{14}{10 + 14} \right) \times 12 = 7 \text{ volts}$$

To calculate the Thévenin's equivalent resistance redraw the circuit with all voltage sources as shorts and calculate the resistance looking into the terminals. Figure 1c shows the circuit as it should be redrawn. The Thévenin's equivalent resistance is just R_1 and R_2 in parallel or 5.83 ohms. Using equation 5 from last month:

$$R_{Total} = \frac{R_1 \times R_2}{R_1 + R_2} \quad (5)$$

$$R_{Total} = \frac{10 \times 14}{10 + 14} = 5.83\Omega$$

Last Month's Problem Three:

Thévenin's Theorem is a very powerful tool for simplifying circuits. Let's look at problem 3 from last month. The circuit is redrawn as figure 2a. You probably had a lot of trouble solving it. Assuming you didn't get any help from Thévenin and you are a math wiz, you most likely solved it by using simultaneous equations (three equations and three unknowns); it was not an easy task! Since we want to find the current through R_3 , let's replace it with two terminals as shown in figure 2b. This is just a series circuit with two batteries and two resistors. Since the voltage at point A is +20V and the voltage at point B is +10V there is 20V minus 10V across the two series resistors R_1 and R_2 . The circuit current through these resistors is:

$$I = \frac{E}{R} = \frac{(E_2 - E_1)}{(R_1 + R_2)}$$

$$I = \frac{(20 - 10)}{(10 + 10)} = 0.5 \text{ amps}$$

And the voltage at the upper terminal is the voltage at point A minus the voltage drop across R_2 , which is 5 volts:

$$E_{R2} = I \times R = 0.5 \times 10 = 5 \text{ volts}$$

$$E_{Th} = 20 - 5 = 15 \text{ volts}$$

The Thévenin's equivalent resistance is easily determined by redrawing the circuit with the two batteries replaced with a short. It is just the two 10 ohm resistors in parallel or 5 ohms. Figure 3a is the Thévenin's equivalent circuit of figure 2b. Now, to solve problem 3, let's just add R_3 , the resistor we removed earlier, across the terminals as shown in figure 3b and solve for the current:

$$I = \frac{E}{R} = \frac{E_{Th}}{(R_{Th} + R_3)}$$

$$I = \frac{15}{5 + 5} = 1.5 \text{ amps.}$$

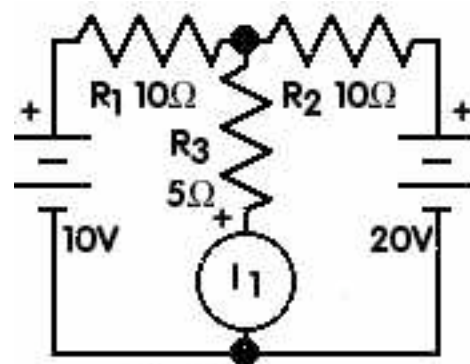


Fig. 2a

Problem #3 from TechTalk 11.

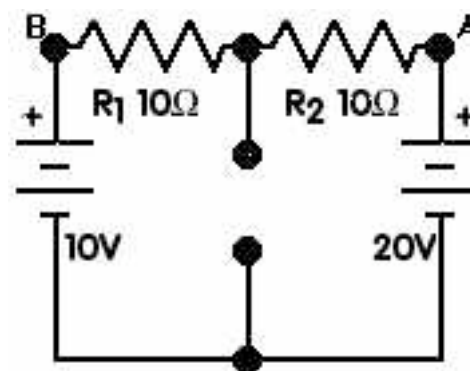
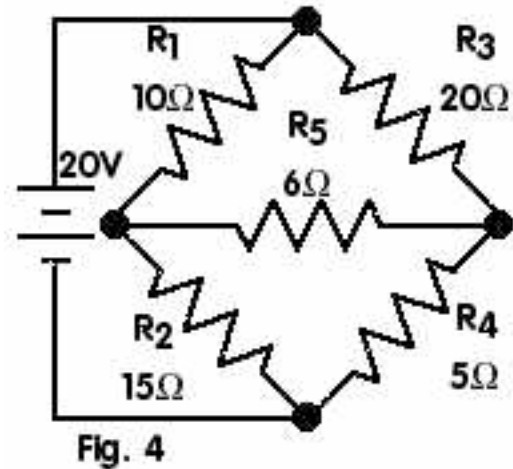
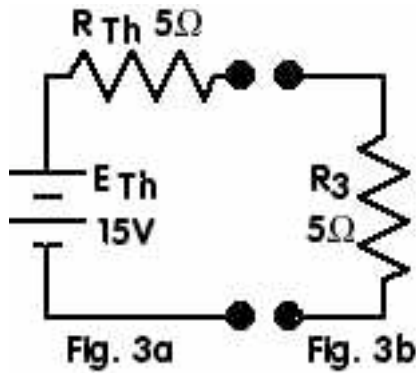


Fig. 2b

Circuit with R_3 and the meter removed, ideal for making a Thévenin equivalent circuit

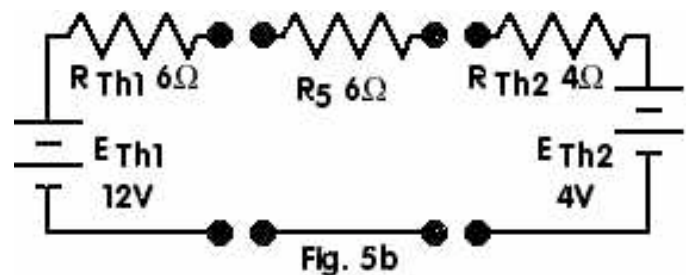
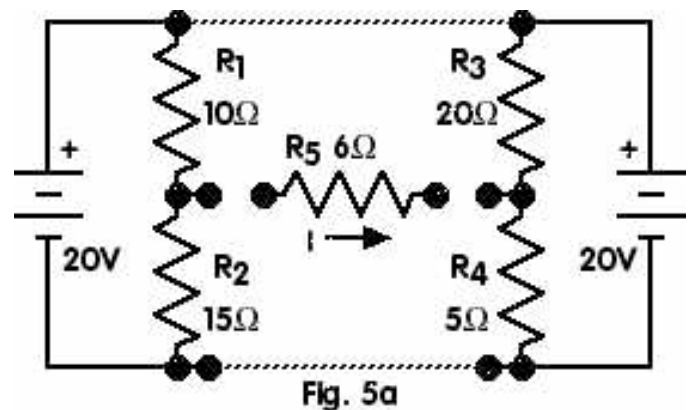


The Wheatstone Bridge:

Let's try another one! The Wheatstone bridge circuit, shown in figure 4, is used with strain gages, instrumentation transducers and measurement test equipment. Basically, when the ratio of R_1 to R_2 is equal to the ratio of R_3 to R_4 the bridge is balanced and no current flows through R_5 , which is often a sensitive meter or other detector. The values given in figure 4 are for an unbalanced bridge so current is flowing through R_5 . Our assignment is to find out what the value of that current is. Those resistors at 45 degree angles have always looked overwhelming, so our first task will be to redraw the circuit as shown in figure 5a. Note that I've broken the circuit into two voltage dividers with their Thévenin terminals, and separated out R_5 . I've also shown the battery twice so it appears on both sides of the circuit, which changes nothing as shown by the dotted circuit lines. Looking at the left side voltage divider, the Thévenin's equivalent voltage and resistance are calculated using equation (6) and equation (5) respectively as:

$$E_{Th1} = \left(\frac{15}{10 + 15} \right) \times 20 = 12 \text{ volts}$$

$$R_{Th1} = \frac{(10 \times 15)}{(10 + 15)} = 6 \text{ ohms}$$



Looking at the right side voltage divider, the Thévenin's equivalent voltage and resistance are:

$$E_{Th2} = \left(\frac{5}{20 + 5} \right) \times 20 = 4 \text{ volts}$$

$$R_{Th2} = \frac{(20 \times 5)}{(20 + 5)} = 4 \text{ ohms}$$

The Wheatstone circuit of figure 4 can now be drawn as figure 5b, and the current through R_5 can be easily calculated:

$$I_{R_5} = \frac{E}{R} = \frac{E_{Th1} - E_{Th2}}{R_{Th1} + R_{Th2} + R_5}$$

$$I_{R_5} = \frac{12 - 4}{6 + 4 + 6} = 0.5 \text{ amps}$$

This is a good example of how Thévenin's Theorem can simplify an otherwise difficult circuit problem. While we haven't probed too deeply, you should now have a good understanding of Ohm's law and how to use it in basic and even somewhat complex circuits. There is another tool, Norton's Theorem that can also be used to simplify circuits. It is beyond the scope of this month's article but it may get discussed in a future issue if anyone shows interest (hint)!

Non-Ideal Meters:

Last month we talked about ideal voltmeters (infinite resistance) and ammeters (zero resistance). Unfortunately these are only available in the labs at *Hogwart's School of Witchcraft and Wizardry*. As a "muggle" you must use non-ideal meters. Let's look at ammeters first:

Analog ammeters have a low series resistance that varies with range. A standard 0-1mA meter typically has a series resistance of 50 to 100 ohms. By Ohm's law this amounts to a voltage drop of 50 to 100 mV across the meter. In certain applications this can have a noticeable influence on the circuit being measured. The series resistance does provide one benefit; by adding a resistor in parallel with the meter, the meter can be made to operate at a higher full-scale value. This resistor is called a meter shunt. If you want to have a 0-1 mA meter with a series resistance of 50 ohms read 0-5 mA full-scale,

a shunt of 12.5 ohms can be placed across the meter. This is also a simple Ohm's law problem!

Ammeters are normally manufactured in the range of 20 uA full-scale to 5 amps full-scale; very low current ammeters used in labs are called galvanometers. The full-scale range of high current ammeters is often 5 amps and either an internal or external shunt is used. For very large currents the external shunt can be placed in a convenient place and the wires to the meter need only handle the lower current.

Non-Ideal Voltmeters:

If you place a resistor in series with a 0-1 mA ammeter and select the resistor so that the total resistance of the meter and resistor in series is 10,000 ohms you have a voltmeter that will read 10 volts full-scale. Other resistances may be calculated for other full-scale voltage ranges using Ohm's law. For large resistors the small meter resistance can be ignored. The example above has 0.5% error.

This voltmeter draws one milliamp from the circuit being measured. It can have a major influence on a high impedance circuit. Since the voltmeter described above has a 10 volt full-scale range and a 10,000 ohm impedance, it is said to be a 1000 "ohm-per-volt" meter. Common values of 1000, 20,000 and 30,000 "ohms per volt" corresponding to meter movements of 1mA, 50 uA and 33uA. The higher the value the less influence on the circuit. To find the resistance of a voltmeter multiply the "ohms per volt" by the full-scale value of the meter (If it's a multirange meter, use the full scale value of the range you're on.) For an old workhorse Simpson Model 260 Volt-Ohm Meter (20,000 ohms per volt) the meter resistance is 50,000 ohms on the 2.5 volt scale and 20 Meg-ohm on the 1000 volt scale. Vacuum tube and FET voltmeters

were developed to allow a higher meter impedance at the lower voltage ranges and thus influence the circuit under test less. Most Heathkit VTVMs have an impedance of 11 Meg ohms on all ranges!

Meters would make an interesting topic for a future Tech Talk; but for the next installment we will continue with Ohm's law and discuss Ohm's power law. Resistive devices dissipate power; this law calculates the power being dissipated in the resistance or load.

Best wishes for a great Holiday Season to your family from AF6C!

73, from AF6C



This article is based on the TechTalk article that originally appeared in the December Year issue of RF, the newsletter of the Orange County Amateur Radio Club - W6ZE.